

# Kaon semileptonic decay ( $K_{l3}$ ) form factor in the nonlocal chiral quark model

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## Abstract

We investigate the kaon semileptonic decay ( $K_{l3}$ ) form factors within the framework of the nonlocal chiral quark model ( $\chi$ QM) from the instanton vacuum, taking into account the effects of flavor SU(3) symmetry breaking. All theoretical calculations are carried out without any adjustable parameter. We also show that the present results satisfy the Callan-Treiman low-energy theorem as well as the Ademollo-Gatto theorem. It turns out that the effects of flavor SU(3) symmetry breaking are essential in reproducing the kaon semileptonic form factors. The present results are in a good agreement with experiments, and are compatible with other model calculations.

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## I. INTRODUCTION

It is of great importance to understand semileptonic decays of kaons ( $K_{l3}$ ), since it plays a significant role in determining the CKM matrix element  $|V_{us}|$  precisely [1]. The effect of flavor SU(3) symmetry breaking on the kaon semileptonic decay form factor is known to be around  $3 \sim 5\%$ , which is rather small. The well-known soft-pion Callan-Treiman [2] theorem connects the ratio of the pion and kaon decay constants to the semileptonic form factors of the kaon at  $q^2 = m_K^2 - m_\pi^2$  (Callan-Treiman point). Experimentally, there are a certain amount of data to judge theoretical calculations [3]. Thus, the kaon semileptonic decay form factor provides a basis to examine the validity and reliability of any theoretical theory and model for hadrons. Related works on the kaon semileptonic decay can be found in Refs. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15],

In the present work, we will investigate the  $K_{l3}$  form factor within the framework of the nonlocal chiral quark model ( $\chi$ QM) derived from the instanton vacuum. We will consider the leading order in the large  $N_c$  expansion and flavor SU(3) symmetry breaking explicitly. The model has several virtues: All relevant QCD symmetries are satisfied within the model, and there are only two: The average size of instantons ( $\rho \sim 1/3$  fm) and average inter-instanton distance ( $R \sim 1$  fm), which can be determined by the internal constraint such as the self-consistent equation [16, 17]. There is no further adjustable parameter in the model.

We employ the modified low-energy effective partition function with flavor SU(3) symmetry breaking [18]. This partition function extends the former one derived in the chiral limit [17]. It has been proven that the partition function with flavor SU(3) symmetry breaking is very successful in describing the low-energy hadronic properties such as various QCD condensates, magnetic susceptibilities, meson distribution amplitudes, and so on [19, 20, 21]. However, the presence of the nonlocal interaction between quarks and pseudo-Goldstone bosons breaks the Ward-Takahashi identity for the Nöther currents. Since the kaon semileptonic decay form factors involve the vector current, we need to deal with this problem. Thus, in the present work, we will investigate the kaon semileptonic decay ( $K_{l3}$ ) form factors, using the gauged low-energy effective partition function from the instanton vacuum with flavor SU(3) symmetry breaking explicitly taken into account.

## II. FORMALISMS

In the present work, we are interested in the following kaon semileptonic decays ( $K_{l3}$ ) in two different isospin channels:

$$\begin{aligned} K^+(p_K) &\rightarrow \pi^0(p_\pi) l^+(p_l) \nu_l(p_\nu) : K_{l3}^+, \\ K^0(p_K) &\rightarrow \pi^-(p_\pi) l^+(p_l) \nu_l(p_\nu) : K_{l3}^0, \end{aligned} \quad (1)$$

where  $l$  and  $\nu_l$  stand for the leptons (either the electron or the muon) and neutrinos. The decay amplitude ( $T_{K \rightarrow l\nu\pi}$ ) can be expressed as follows [8]:

$$T_{K \rightarrow l\nu\pi} = \frac{G_F}{\sqrt{2}} \sin \theta_c [w^\mu(p_l, p_\nu) F_\mu(p_K, p_\pi)], \quad (2)$$

where  $G_F$  is the well-known Fermi constant ( $1.166 \times 10^{-5} \text{ GeV}^{-2}$ ).  $\theta_c$  denotes the Cabbibo angle. We define respectively the weak leptonic current ( $w^\mu$ ) and hadronic matrix element

$F_\mu$  with the  $\Delta S = 1$  vector current ( $j_\mu^{su}$ ) as:

$$w^\mu(p_l, p_\nu) = \bar{u}(p_\nu)\gamma^\mu(1 - \gamma_5)v(p_l), \quad (3)$$

$$\begin{aligned} F_\mu(p_K, p_\pi) &= c\langle\pi(p_\pi)|j_\mu^{su}|K(p_K)\rangle = c\langle\pi(p_\pi)|\bar{\psi}\gamma_\mu\lambda^{4+i5}\psi|K(p_K)\rangle \\ &= (p_K + p_\pi)_\mu f_{l+}(t) + (p_K - p_\pi)_\mu f_{l-}(t), \end{aligned} \quad (4)$$

where  $c$  is the isospin factor, and set to be unity and  $1/\sqrt{2}$  for  $K_{l3}^0$  and  $K_{l3}^+$ , respectively. The matrix  $\lambda^{4+i5}$  denotes the combination of the two Gell-Mann matrices,  $(\lambda^4 + i\lambda^5)/2$ , for the relevant flavor in the present problem. The  $\psi$  denotes the quark field. The momentum transfer is defined as  $Q^2 = (p_K - p_\pi)^2 \equiv -t$ .

$f_{l\pm}$  represent the vector form factors with the corresponding lepton  $l$  ( $P$ -wave projection). Alternatively, the form factor  $F_\mu(p_K, p_\pi)$  can be expressed in terms of the scalar ( $f_{l0}$ ,  $S$ -wave projection) and the vector form factor  $f_{l+}$  defined as follows:

$$F_\mu(p_K, p_\pi) = f_{l+}(t)(p_K + p_\pi)_\mu + \frac{(m_\pi^2 - m_K^2)(p_K - p_\pi)_\mu}{t} [f_{l+}(t) - f_{l0}(t)]. \quad (5)$$

Hence, the  $f_{l0}$  can be written as the linear combination of  $f_{l+}$  and  $f_{l-}$ :

$$f_{l0}(t) = f_{l+}(t) + \left[ \frac{t}{m_K^2 - m_\pi^2} \right] f_{l-}(t). \quad (6)$$

Since the isospin breaking effects are almost negligible, we will consider only the  $K^0 \rightarrow \pi^- \nu l^+$  decay channel.

It has been well-known that the experimental data for  $f_{l+,0}$  can be reproduced qualitatively well by the linear and quadratic fits [3]:

$$\begin{aligned} \text{Linear : } f_{l+,0}(t) &= f_{l+,0}(0) \left[ 1 + \frac{\lambda_{l+,0}}{m_\pi^2} (t - m_l^2) \right], \\ \text{Quadratic : } f_{l+,0}(t) &= f_{l+,0}(0) \left[ 1 + \frac{\lambda'_{l+,0}}{m_\pi^2} (t - m_l^2) + \frac{\lambda''_{l+,0}}{2m_\pi^4} (t - m_l^2)^2 \right], \end{aligned} \quad (7)$$

where  $m_l$  is the lepton mass. The slope parameter  $\lambda_{l+}$  has an important physical meaning. For example, the  $K \rightarrow \pi$  decay radius  $(\langle r^2 \rangle^{K\pi})$  can be obtained as follows [8]:

$$\lambda_+ \simeq \frac{1}{6} \langle r^2 \rangle^{K\pi} m_\pi^2. \quad (8)$$

Moreover, this radius is also related to the Gasser-Leutwyler low-energy constant  $L_9$  in the large  $N_c$  limit [6] as follows:

$$L_9 = \frac{1}{12} F_\pi^2 \langle r^2 \rangle^{K\pi}. \quad (9)$$

We now show how to derive the hadronic matrix element given in Eq. (4) within the framework of the nonlocal  $\chi$ QM from the instanton vacuum. We begin by the low-energy effective QCD partition function derived from the instanton vacuum [18]:

$$\begin{aligned} \mathcal{Z}_{\text{eff.}} &= \int \mathcal{D}\psi \mathcal{D}\psi^\dagger \mathcal{D}\mathcal{M} \exp \int d^4x \left[ \psi_f^\dagger(x) (i\not{\partial} + im_f) \psi_f(x) \right. \\ &\quad \left. + i \int \frac{d^4k d^4p}{(2\pi)^8} e^{-i(k-p)\cdot x} \psi_f^\dagger(k) \sqrt{M_f(k_\mu)} U_{fg}^{\gamma_5} \sqrt{M_g(p_\mu)} \psi_g(p) \right]. \end{aligned} \quad (10)$$

$M_f(k)$  is the dynamically generated quark mass being momentum-dependent, whereas  $m_f$  stands for the current-quark mass with flavor  $f$ .  $U^{\gamma_5}$  is the nonlinear background Goldstone boson field. As mentioned previously, the momentum-dependent dynamical quark mass  $M_f(k)$  breaks the conservation of the Nöther (vector) currents. Refs. [19] derived the light-quark partition function in the presence of the external vector field. By doing so, we can derive the gauge-invariant formula for the kaon semileptonic form factor as follows:

$$\begin{aligned}
F_\mu^{\text{local(a)}} &= \frac{8N_c}{F_\pi F_K} \int \frac{d^4k}{(2\pi)^4} \frac{M_q(k_a) \sqrt{M_s(k_b) M_q(k_c)}}{[k_a^2 + \bar{M}_q^2(k_a)] [k_b^2 + \bar{M}_s^2(k_b)] [k_c^2 + \bar{M}_q^2(k_c)]} \\
&\times \left[ [k_a \cdot k_b + \bar{M}_q(k_a) \bar{M}_s(k_b)] k_{c\mu} - [k_b \cdot k_c + \bar{M}_s(k_b) \bar{M}_q(k_c)] k_{a\mu} \right. \\
&\left. + [k_a \cdot k_c + \bar{M}_q(k_a) \bar{M}_q(k_c)] k_{b\mu} \right], \tag{11}
\end{aligned}$$

where  $\bar{M}_f(k) = m_f + M_f(k)$ . The relevant momenta are defined as  $k_a = k - p/2 - q/2$ ,  $k_b = k + p/2 - q/2$  and  $k_c = k + p/2 + q/2$ , in which  $k$ ,  $p$  and  $q$  denote the internal quark, initial kaon, and transfered momenta, respectively. The trace  $\text{tr}_\gamma$  runs over Dirac spin space. Similarly, we can evaluate the nonlocal contributions as follows [20]:

$$\begin{aligned}
F_\mu^{\text{nonlocal(b)}} &= \frac{8N_c}{F_\pi F_K} \int \frac{d^4k}{(2\pi)^4} \frac{\sqrt{M_q(k_c)}_\mu \sqrt{M_q(k_c) M_q(k_a) M_s(k_b)}}{[k_a^2 + \bar{M}_q^2(k_a)] [k_b^2 + \bar{M}_s^2(k_b)] [k_c^2 + \bar{M}_q^2(k_c)]} \\
&\times [\bar{M}_q(k_c) k_a \cdot k_b + \bar{M}_s(k_b) k_a \cdot k_c - \bar{M}_q(k_a) k_b \cdot k_c + \bar{M}_q(k_a) \bar{M}_s(k_b) \bar{M}_q(k_c)] \\
&- (b \leftrightarrow c), \\
F_\mu^{\text{nonlocal(c)}} &= -\frac{4N_c}{F_\pi F_K} \int \frac{d^4k}{(2\pi)^4} \\
&\times \frac{\sqrt{M_q(k_a)} \sqrt{M_s(k_b)} \sqrt{M_q(k_c)}_\mu \sqrt{M_q(k_a)} [k_a \cdot k_b + \bar{M}_q(k_a) \bar{M}_s(k_b)]}{[k_a^2 + \bar{M}_q^2(k_a)] [k_b^2 + \bar{M}_s^2(k_b)]} \\
&+ \frac{4N_c}{F_\pi F_K} \int \frac{d^4k}{(2\pi)^4} \\
&\times \frac{\sqrt{M_q(k_a)} \sqrt{M_s(k_b)} \sqrt{M_q(k_c)} \sqrt{M_q(k_a)}_\mu [k_a \cdot k_b + \bar{M}_q(k_a) \bar{M}_s(k_b)]}{[k_a^2 + \bar{M}_q^2(k_a)] [k_b^2 + \bar{M}_s^2(k_b)]} \\
&+ (b \leftrightarrow c), \tag{12}
\end{aligned}$$

where  $\sqrt{M(k)}_\mu = \partial \sqrt{M(k)} / \partial k_\mu$ .

### III. NUMERICAL RESULTS

We now discuss various numerical results for the kaon semileptonic decay ( $K_{l3}$ ) form factors in the present work. We facilitate the Breit-momentum framework for convenience by virtue of the Lorentz invariance of the model. We first consider the case of  $K_{e3}$ . In the left panel of Figure 1, we draw the numerical results for  $f_{e+}(t)$  (solid),  $f_{e-}(t)$  (dotted) and  $f_{e0}(t)$

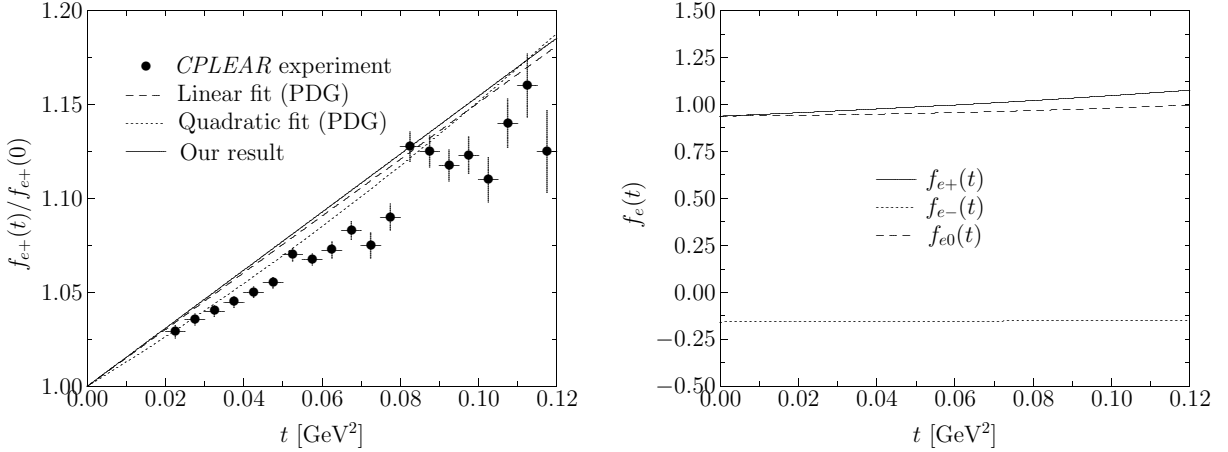


FIG. 1:  $K_{e3}$  form factors,  $f_{e+}(t)$  (solid),  $f_{e-}(t)$  (dotted) and  $f_{e0}(t)$  (dashed) are shown in the left panel, while in the right panel the ratio of  $f_{e+}(t)$  and  $f_{e+}(0)$  is given (solid).

(dashed). Note that the scalar form factor  $f_{e0}(t)$  is derived by using Eq. (6). We observe that the  $f_{e+}(t)$  and  $f_{e0}(t)$  are almost linearly increasing functions of  $t$ , whereas  $f_{e-}(t)$  decreases. At  $t = 0$ , our results demonstrate that  $f_{e+}(0) = f_{e0}(0) = 0.947$  and  $f_{e-}(0) = -0.137$ . In the chiral limit,  $f_{e+}(0)$  and  $f_{e-}(0)$  should be unity and zero, respectively, which is related to the Ademollo-Gatto theorem in the case of pseudo-Goldstone bosons [6, 22, 23]:

$$\lim_{q \rightarrow 0} F_{\mu}^{\text{local(a)}} \simeq 2p_{\mu} + \mathcal{O}(m_q). \quad (13)$$

The Ademollo-Gatto theorem in Eq. (13) can be easily tested in the nonlocal  $\chi$ QM. Considering  $q \rightarrow 0$  and ignoring the terms being proportional to  $k \cdot p$ , the leading contribution of Eq. (11) can be rewritten upto  $\mathcal{O}(m_q)$  as follows:

$$\lim_{q \rightarrow 0} F_{\mu}^{\text{local(a)}} \simeq 2[1 + R(m_s)]p_{\mu}, \quad (14)$$

where

$$R(m_s) = \frac{1}{2} \left[ \int \frac{d^4k}{(2\pi)^4} \frac{M^2(k)m_s[m_s + 2M(k)]}{[k^2 + M^2(k)]^3} \right] \left[ \int \frac{d^4k}{(2\pi)^4} \frac{M^2(k)}{[k^2 + M^2(k)]^2} \right]^{-1}. \quad (15)$$

To evaluate Eq. (14), we employ the ratio  $F_K/F_{\pi}$  computed within the same framework and expanded in terms of the strange quark mass ( $m_s$ ):

$$\frac{F_K}{F_{\pi}} \simeq 1 + R(m_s). \quad (16)$$

We also use that  $k_b = k_c \rightarrow k + p/4$  since these two momenta share  $p/2$  as  $q \rightarrow 0$ . Note that we consider only the local contribution for  $F_{\mathcal{M}}$  in Eq. (16). We, however, verified that the nonlocal contributions in Eq. (12) also satisfies the Ademollo-Gatto theorem analytically.

In the right panel of Figure 1 we draw the ratio of  $f_{e+}(t)$  and  $f_{e+}(0)$  with respect to the CPLEAR experimental data [24], and linear (dashed) and quadratic (dotted) fits for the ratio using the PDG data [3]:  $\lambda_{e+} = (2.960 \pm 0.05) \times 10^{-2}$ ,  $\lambda'_{e+} = (2.485 \pm 0.163) \times 10^{-2}$ , and

$\lambda''_{e+} = (1.920 \pm 0.062) \times 10^{-3}$ . In the present calculation, we obtain  $\lambda_{e+} = 3.028 \times 10^{-2}$  for the linear fit, which is very close to the experimental one,  $2.960 \times 10^{-2}$ . Since our result for  $f_{e+}$  is almost linear as shown in Fig. 1, we get almost a negligible value for the slope parameter  $\lambda''$  when the quadratic fit is taken into account. Being compared with other model calculations, the present results are comparable to those from  $\chi$ PT [8], and other models [9, 11, 12, 25]. Using Eq. (8) and Eq. (9), we can easily estimate the  $K_{e3}$  decay radius and low-energy constant  $L_9$ , respectively. As for the  $K_{e3}$  decay radius, we obtain  $\langle r^2 \rangle^{K\pi} = 0.366 \text{ fm}^2$ . This value is slightly larger than that in  $\chi$ PT [6]. The low-energy constant  $L_9$  turns out to be  $6.78 \times 10^{-3}$ , which is comparable to  $7.1 \sim 7.4 \times 10^{-3}$  [6] and  $6.9 \times 10^{-3}$  [8, 26].

The ratio of the pion and kaon weak decay constants  $F_K/F_\pi$  can be deduced from the scalar form factor  $f_0$  via the Callan-Treiman soft-pion theorem [2]. In the soft-pion limit ( $p_\pi \rightarrow 0$ ), the  $K_{e3}$  form factor can be written as [27, 28, 29]:

$$\lim_{p_\pi \rightarrow 0} F_\mu(p_\pi, p_K) = p_{K\mu} \frac{F_K}{F_\pi}. \quad (17)$$

Using Eqs. (4) and (6), we obtain the following expression:

$$\lim_{p_\pi \rightarrow 0} F_\mu(p_\pi, p_K) = \lim_{p_\pi \rightarrow 0} (p_\pi + p_K)_\mu [f_{l+}(\Delta_{\text{CT}}) + f_{l-}(\Delta_{\text{CT}})] \simeq p_{K\mu} f_{l0}(\Delta_{\text{CT}}), \quad (18)$$

where the value of  $\Delta_{\text{CT}} = m_K^2 - m_\pi^2$  is called the Callan-Treiman point which can not be accessible physically. Combining Eq. (17) with Eq.(18), we finally arrive at the final expression of the  $K_{l3}$  form factor for the Callan-Treiman theorem in terms of the scalar form factor and the ratio,  $F_K/F_\pi$ :

$$f_{e0}(m_K^2 - m_\pi^2) = \frac{F_K}{F_\pi}. \quad (19)$$

From our numerical calculation using Eq. (19) we find that  $F_K/F_\pi = 1.08$ , which is  $\sim 10\%$  smaller than the empirical value (1.22). This smallness is mainly depends on the nonlocal contributions (c) in Eq. (12) such that  $f_{e-}$  decrease as depicted in the left panel of Figure 1. This behavior can be interpreted by the fact that the kaon weak decay constant turns out to be smaller if we ignore the meson-loop correction in the nonlocal  $\chi$ QM [30] and in chiral perturbation theory ( $\chi$ PT) as well, in which the ratio is defined in the large  $N_c$  limit by:

$$\frac{F_K}{F_\pi} = 1 + \frac{4}{F_\pi^2} (m_K^2 - m_\pi^2) L_5. \quad (20)$$

Using the value of  $F_K/F_\pi = 1.08$ , we obtain  $L_5 = 7.67 \times 10^{-4}$  which is underestimated by a half of the phenomenological one  $1.4 \times 10^{-3}$  [26]. It is well known that in order to reproduce the  $L_5$  within the  $\chi$ QM the meson-loop  $1/N_c$  corrections are essential.

In the soft limit, the model should satisfy the Callan-Treiman theorem given in Eq. (19). Taking the limit  $p_\pi \rightarrow 0$  for Eq. (11), we can show that Eq. (11) satisfies the Callan-Treiman theorem using Eq. (16) as follows:

$$\lim_{p_\pi \rightarrow 0} F_\mu^{\text{local(a)}} \simeq [1 + R(m_s)] p_\mu, \quad (21)$$

where  $k_a = k_c \rightarrow k$  as  $p_\pi \rightarrow 0$ . Inserting Eq. (16) into Eq. (21), we can verify the validity of the Callan-Treiman theorem in Eq. (17) (Eq. (19)). The same argument also holds for the nonlocal contributions.

The decay width of  $K \rightarrow \pi \nu e$  can be easily computed by using the result of  $f_{l+,0}$ . It turns out that  $\Gamma_{e3} = 6.840 \times 10^6/\text{s}$  and  $\Gamma_{\mu 3} = 4.469 \times 10^6/\text{s}$  with  $|V_{us}| = 0.22$  taken into account [3, 31]. The results are slightly smaller than the experimental data ( $\Gamma_{e3} = (7.920 \pm 0.040) \times 10^6/\text{s}$  and  $\Gamma_{\mu 3} = (5.285 \pm 0.024) \times 10^6/\text{s}$ ) [3].

## IV. SUMMARY AND CONCLUSION

In the present work, we have investigated the kaon semileptonic decay ( $K_{l3}$ ) form factors within the framework of the gauged nonlocal chiral quark model from the instanton vacuum. The effect of flavor SU(3) symmetry breaking were taken into account. We calculated the vector form factors ( $f_{\pm}$ ), scalar form factor ( $f_0$ ), slope parameters ( $\lambda_{+,0}$ ), decay width ( $\Gamma_{l3}$ ), etc. We found that the present results of the kaon semileptonic decay form factors are in a qualitatively good agreement with experiments. We emphasize that there were no adjustable free parameters in the present investigation. All results were obtained with only two parameters from the instanton vacuum, i.e. the average instanton size ( $\bar{\rho} \sim 1/3$  fm) and inter-instanton distance ( $R \sim 1$  fm).

In the present investigation, we have considered only the leading-order contributions in the large  $N_c$  limit. While these contributions reproduce the observables relevant for kaon semileptonic decay in general, it seems necessary to take into account the  $1/N_c$  meson-loop corrections in order to reproduce quantitatively the kaon decay constant  $f_K$  and the low-energy constant  $L_5$ . As noticed in Refs. [30, 32, 33], this correction for the fluctuation (meson-loop correction) can play an important role in producing the kaon properties as shown in the ratio  $F_K/F_{\pi}$  as discussed and showed in the text. Moreover, it was shown that some of the low energy constants are very sensitive to this correction. Related works are under progress. For more details on the present work, one can refer to Ref. [34].

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